SOLUTION BY ARTEMAS MARTIN, M. A., ERIE, PA.

Assume $x = an^2 - 2mn$, $y = m^2 - n^2$, $z = cn^2 + 2mn$; then

$$x^2 + axy + y^2 = (m^2 - amn + n^2)^2,$$

$$y^2 + cyz + z^2 = (m^2 + cmn + n^2)^2.$$

But the second condition is not satisfied by the assumed values of x, y and z.

Let us put

$$an - 2m = r, \tag{1}$$

$$cn + 2m = s; \tag{2}$$

then $x = nr$, $z = ns$, $x^2 + bxz + z^2 = n^2(r^2 + brs + s^2)$.

Assume now

$$r = t(p^2 - q^2), \tag{3}$$

$$s = t(bq^2 + 2pq), \tag{4}$$

and we have $r^2 + brs + s^2 = t^2(p^2 + bpq + q^2)^2$, and consequently

$$x^2 + bxz + z^2 = n^2t^2(p^2 + bpq + q^2)^2.$$

From (1), (2), (3) and (4) we have

$$an - 2m = t(p^2 - q^2), \tag{5}$$

$$cn + 2m = t(bq^2 + 2pq); \tag{6}$$

which give

$$m = \frac{at(bq^2 + 2pq) - ct(p^2 - q^2)}{2(a+c)} = a(bq^2 + 2pq) - c(p^2 - q^2),$$

$$n = \frac{2t(bq^2 + 2pq) + 2t(p^2 - q^2)}{2(a+c)} = 2(bq^2 + 2pq) + 2(p^2 - q^2),$$

if we take $t = 2(a+c)$; where p and q may be any numbers that will give x, y, z all positive or all negative. See *Laybourn's Mathematical Repository*, New Series, Vol. 3, Part I, pp. 151—164, for elaborate general solutions to this problem.

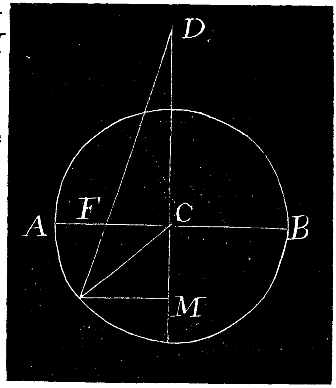
QUERY. "In works on Practical Geometry, the following method of inscribing a regular polygon of n sides is given:

Let AB be a diameter of a circle. With A and B as centers and radius AB describe arcs intersecting in D. Divide AB into n equal parts. Draw DE through the second point of division; the arc AE is one-nth part of the circumference. Is there a general demonstration published?"

ANSWER BY E. B. SEITZ.

The method is strictly correct only for regular polygons of 3, 4 and 6 sides, which is shown as follows:

Let C be the center of the circle, F the second point of division; draw EM perpendicular to DC produced. Let $AC = r$, $CM = x$, and $\angle ACE = \varphi$.



Then $CD = r\sqrt{3}$, $AF = 4r \div n$, $FC = r -(4r \div n)$, and $r\sqrt{3} : x + r\sqrt{3} :: r -(4r \div n) : EM = (n-4)(x\sqrt{3}+3r) \div 3n$; therefore

$$x^2 + \frac{(n-4)^2(x\sqrt{3}+3r)^2}{9n^2} = r^2, \qquad \text{whence}$$

$$x = \frac{nr\sqrt{[3(n^2+16n-32)]}-(n-4)^2 r\sqrt{3}}{4(n^2-2n+4)},$$

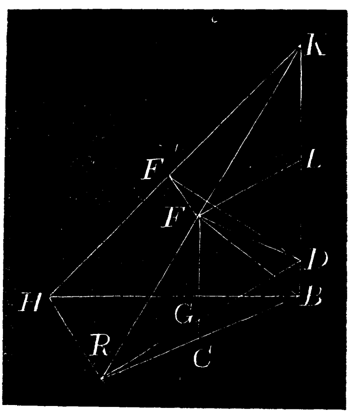
and $$\varphi = \sin^{-1}\left(\frac{x}{r}\right).$$

The following table shows the values of φ from $n=3$ to $n=10$, togather with their errors.

n.	φ.	Error.
3	120°00′ 00″	0′ 00″
4	90 00 00	0 00
5	71 57 12	2 48
6	60 00 00	0 00
7	51 31 05	5 22
8	45 11 15	11 15
9	40 16 39	16 39
10	36 21 21	21 21.

[By request of Mr. Adcock, who contends that the published solution of problem 195 is erroneous, we insert the following solution of that problem by Mr. Adcock. From a hasty examination of both solutions we have not been able to perceive any conflict between them: Mr. Eastwood has determined the normal pressure at both ends of the rod, while Mr. Adcock finds the locus of the upper end.]

Let l = length of the rod RD resting on the hsrizontal plane HBR at R and against the vertical plane HBK at D. $HB = x$, $HR = y$, $BD = z$, then $$x^2+y^2+z^2 = l^2 \qquad (1)$$



Through R and in the vertical plane RBD draw RFK making the angle $CRF = \tan^{-1}(1 \div m)$, and through G, the centre of gravity of the rod, draw the vertical line CGF, join FD, draw FL parallel to RB, and FF' parallel to RH. Then, from the figure, $RC = CB = FL = \frac{1}{2}\sqrt{(x^2+y^2)}$, $CG = \frac{1}{2}BD = \frac{1}{2}z$, $CF = \sqrt{(x^2+y^2)} \div 2m$, $RF = FK = (x^2+y^2)^{\frac{1}{2}} \times (1+m^2)^{\frac{1}{2}} \div 2m$, $2FG = KD = \sqrt{(x^2+y^2)} \div m - z$, $FD = \sqrt{[(FC-BD)^2+BC^2]} = \sqrt{\{[\sqrt{(x^2+y^2)} \div 2m - z]^2 + \frac{1}{4}(x^2+y^2)\}}$,

$FF' = \frac{1}{2}z$, $F'D = \surd(FD^2+FF'^2) = \surd\{[\surd(x^2+y^2)\div 2m - z]^2 + \frac{1}{4}x^2\}$.

Now if $2FG = DK$ represent the weight, w, of the rod, FR will be the resultant of pressure FC and friction RC at R, and FF' and $F'D$ will represent the pressure and friction at D. Hence

$$\frac{F'D}{FF'} = \frac{\surd\{[\surd(x^2+y^2)\div 2m - z]^2 + \frac{1}{4}x^2\}}{\frac{1}{2}z} = m'. \tag{2}$$

Eliminating y from (2) by (1),

$$\frac{[\surd(l^2-z^2)\div 2m - z]^2 + \frac{1}{4}x^2}{l^2-x^2-z^2} = 4m'^2,$$

which is the equation of the locus of the upper end of the rod in all its positions of equilibrium on the supposition that the lower end is always in the axis of y.

PROBLEMS.

211. By W. E. Heal.— Prove that every number is either a triangular number or is the sum of two, or of three triangular numbers.

212. By E. B. Opdycke, Pulaski, Ohio. — One half of a circular tract of land is cut off by an arc of a circle whose center is in the circumference of the circular tract. Find the radius with which the arc is described.

213. By Geo. H. Harvill, Bonner, La. — Upon the three sides of any triangle construct equilateral triangles and join their centers by right lines. Prove that the triangle so formed is equilateral.

214. By G. Shaw, Kemble, Ont., Canada.—Prove that

$$\cfrac{1}{\tan A + \cfrac{1}{\tan A + \&c.}} = \tfrac{1}{2}[\surd(\sec^2 A + 3) - \tan A].$$

215. By Prof. Beman. — A harbor A is so situated with reference to two headlands B and C, that the angle BAC is a right angle. A ship sails in a course making an angle of 55° with AB, to D, when $DB = DC$: she then sails forward on the same course 15 ms. to E, when BEC is a straight line. Required AB, AC, DB, EB and EC.

216. By Prof. Scheffer. Through three given points to describe the maximum ellipse.

217. By Prof. W. W. Hendrickson. —The hypothenuse of a right triangle is fixed, and squares are described upon the other two sides: it is required to find the equation to the locus of the intersection of two straight